JOURNAL OF GUIDANCE, CONTROL, AND DYNAMICS
Vol. 28, No. 5, September–October 2005

Differential Geometric Guidance Based on the Involute of the Target's Trajectory

Omar Ariff,* Rafał Żbikowski,† Antonios Tsourdos,‡ and Brian A. White§
Cranfield University, Swindon, England SN6 8LA, United Kingdom

This paper presents a novel approach to missile guidance using the differential geometry of curves and not relying on the line of sight information. The target's trajectory is treated as a smooth curve of known curvature and the new algorithm is based on the involute of the target's curve. The missile's trajectory uses the concept of virtual target to generate the correct involute trace. It is shown that the missile is either on the trace immediately or may be able to reach it through an alignment procedure. In general, following the trace requires a three-dimensional maneuver in which the missile flies above the target's tangent plane. The projection of the three-dimensional trajectory onto the tangent plane coincides with the involute trace, but is traversed in the time-to-go, thus resulting in the intercept. Two air-to-air scenarios of point masses are considered for a maneuvering target of the F-16 fighter class: 1) a two-dimensional engagement with target executing a constant *g* turn; 2) a three-dimensional engagement with target executing a barrel-roll maneuver. Perfect target information is assumed in simulations. In the first case, intercepts occur both for the involute law and proportional-navigation (PN) guidance; PN based intercepts occur quicker, but the involute-based trajectories are more difficult to evade and always result in a side impact. In the second case, PN fails to intercept the target, while the involute law is successful.

I. Introduction

THIS paper offers a new perspective on the generation of intercept trajectories for missile guidance. The motivation is to overcome, without the complexity of optimal control,[1] the three shortcomings of the proportional-navigation (PN) algorithm[2] family: 1) reliance on the line of sight (LOS) for derivation of intercept geometry and kinematics, 2) effectiveness against nonmaneuvering targets only, and 3) lack of direct control over the curvature of the missile's trajectory. The second limitation can be, to some extent, alleviated through various modifications of the original PN algorithm, for example, augmented PN. However, the LOS motion becomes very rapid when the missile and target are close to each other. This is an inescapable consequence of using straight lines (intercept triangles) to express curved trajectories (maneuvers). Scenarios with maneuvers lead naturally to freeing the intercept geometry of the rectilinear framework by the use of the differential geometry of spatial curves.

The essence of differential geometric description of smooth curves in $\mathbb{R}^3$ is the use of calculus to quantify how a curve deviates locally from its linear approximation. The point-mass velocity vector $\boldsymbol{v} = \boldsymbol{v}(t)$ over time t is expressed as a multiple $v = v(t)$ of the unit tangent vector $\boldsymbol{T} = \boldsymbol{T}(t)$, that is, $\boldsymbol{v}(t) = v(t)\boldsymbol{T}(t)$. At the same moment in time, the normal vector $\boldsymbol{k} = \boldsymbol{k}(t)$ is obtained by differentiating $\boldsymbol{T}$ and is therefore orthogonal to $\boldsymbol{T}$. (Because $\boldsymbol{T}\cdot\boldsymbol{T} = 1$, differentiation gives $\dot{\boldsymbol{T}}\cdot\boldsymbol{T} = 0$.) Again, $\boldsymbol{k}(t) = \kappa(t)\boldsymbol{N}(t)$, where $\boldsymbol{N}$ is the unit normal vector and κ is the curvature. The curvature measures the local deviation of the curve from the rectilinear progression along the tangent line. Indeed, its inverse $\rho = 1/\kappa$ is the radius of best-fitting circle in the plane spanned by the tangent and normal vectors. The right-handed local coordinate frame is completed by the binormal vector $\boldsymbol{b}(t) = \tau(t)\boldsymbol{B}(t)$, where $\boldsymbol{B} \overset{\text{def}}{=} \boldsymbol{T}\times\boldsymbol{N}$ is the unit binormal vector. The torsion τ measures locally how much the curve deviates from the plane spanned by the tangent and normal vectors.

It is convenient to replace the time parameter t with the arc length

$$s \overset{\text{def}}{=} \int_0^t v(t)\,\mathrm{d}t$$

for then the reparameterized curve has unit speed, as $\mathrm{d}s/\mathrm{d}t = v$. The preceding definitions and relationships are summarized[3] by the Frenet–Serret equations:

$$\begin{bmatrix}\boldsymbol{T}'(s)\\ \boldsymbol{N}'(s)\\ \boldsymbol{B}'(s)\end{bmatrix} = \begin{bmatrix}0 & \kappa(s) & 0\\ -\kappa(s) & 0 & \tau(s)\\ 0 & -\tau(s) & 0\end{bmatrix}\begin{bmatrix}\boldsymbol{T}(s)\\ \boldsymbol{N}(s)\\ \boldsymbol{B}(s)\end{bmatrix} \tag{1}$$

where $'$ means differentiation with respect to s. (Note that each of $\boldsymbol{T}$, $\boldsymbol{N}, \boldsymbol{B}$ is a vector in $\mathbb{R}^3$, in general.) Parameterizations of Eq. (1) differ by the factor of speed $v = \mathrm{d}s/\mathrm{d}t$; as for any vector $\boldsymbol{x}$, we obviously have $\mathrm{d}\boldsymbol{x}/\mathrm{d}t = (\mathrm{d}\boldsymbol{x}/\mathrm{d}s)(\mathrm{d}s/\mathrm{d}t)$.

The work[4–6] is a recent example of the use of differential geometry for guidance. In those papers the three-dimensional kinematics of missile-target point masses were resolved with respect to the LOS. The missile trajectory arc length s_m was used as the common parameter for both curves. The target/missile speed ratio $m = v_t/v_m = \text{const} < 1$ was assumed, together with $v_m = \text{const}$. The resulting kinematics formulas combined the unit vector $\boldsymbol{e}_r$ along the LOS and the unit rotational vector $\boldsymbol{e}_\theta$ of the LOS with Eq. (1), so that the vectors $\boldsymbol{T}_t$, $\boldsymbol{T}_m$ (with the speed ratio m) and $\boldsymbol{N}_t$, $\boldsymbol{N}_m$ (with the curvatures κ_t, κ_m) were involved. The end result was a generalization of PN for maneuvering (but constant speed) targets, valid for some initial conditions.

Previous noteworthy work includes the kappa guidance,[7–9] which builds upon the classical work on the E guidance by Cherry.[10] The original algorithm of Lin[7] was two-dimensional and based on the simple reasoning of Cherry that in inertial coordinates the missile acceleration is $\ddot{\boldsymbol{x}} = \boldsymbol{g} + \boldsymbol{a}_c$, where $\boldsymbol{g}$ is the gravity vector and $\boldsymbol{a}_c$ the command vector. The unknown acceleration $\ddot{\boldsymbol{x}}$ can be expanded in a Taylor series, but only two coefficients can be determined, K_1 and K_2. Lin's generalization of Cherry's approach[7] consisted in

Received 21 May 2004; revision received 8 October 2004; accepted for publication 18 October 2004.

*Ph.D. Student, Department of Aerospace, Power and Sensors, Royal Military College of Science, Shrivenham.

†Principal Research Officer, Department of Aerospace, Power and Sensors, Royal Military College of Science, Shrivenham; R.W.Zbikowski@cranfield.ac.uk.

‡Lecturer, Department of Aerospace, Power and Sensors, Royal Military College of Science, Shrivenham.

§Professor, Head of the Department, Department of Aerospace, Power and Sensors, Royal Military College of Science, Shrivenham.

recalculation of K_1 and K_2 via optimal control, resulting from maximization of the final (impact) speed. The end result was suboptimal to achieve analytical formulas, and the whole setting was based on the LOS information. The same was also true in three dimensions, when a torsion command was included.[9]

This paper differs from prior work in two main aspects. First, it focuses on deriving an algorithm for two-dimensional intercept trajectory generation that is not based on a LOS framework. Second, an explicit aim is to tackle maneuvering targets in a way that leads to a conceptually and computationally simple guidance law.

II. Involutes

A curve C is a (thrice) differentiable mapping $x\colon [0, c] \to \mathbb{R}^3$, and its trace is the set $\mathcal{T}(\boldsymbol{x}) = \{\boldsymbol{x}(s) \in \mathbb{R}^3 \mid s \in [0,\ c]\}$. The tangent line to a curve C generates the tangent surface that for planar curves coincides with the defining plane. A curve C^* that lies on the tangent surface of C and intersects the tangent lines orthogonally is called an involute of C (see Fig. 1). The equation of an involute $\boldsymbol{x}^*$ of the curve $\boldsymbol{x}$, is

$$\boldsymbol{x}^*(s) = \boldsymbol{x}(s) + (c - s)\boldsymbol{T}(s) \tag{2}$$

so that $\boldsymbol{x}^*(c) = \boldsymbol{x}(c)$ is the intercept for $s = c$ and

$$\boldsymbol{v}^*(s) = (c - s)\kappa \boldsymbol{N}(s) \tag{3}$$

Here $\boldsymbol{v}^* \overset{\text{def}}{=} \mathrm{d}\boldsymbol{x}^*/\mathrm{d}s$, while s, $\boldsymbol{T}(s)$, $\boldsymbol{N}(s)$, and κ are parameters of the original curve C. This is the kinematic model for involute-based missile-target engagements.

The involute's curvature is expressed in terms of the original curve's curvature κ and torsion τ:

$$(\kappa^*)^2 = \frac{\kappa^2 + \tau^2}{(c - s)^2 \kappa^2} \tag{4}$$

For two dimensions, $\tau = 0$, and in our case $s = vt$, so that

$$\kappa^* = 1/(c - vt) \tag{5}$$

Reparameterization of the preceding formulas with missile arc length $s_m = v_m t = (v_m/v)s$ is obvious, as $v_m = \text{const}$.

III. Involute-Based Guidance Law

There are three main problems in application of the mathematical principles of involutes to practical guidance:

1) Position: The missile's position $\boldsymbol{x}^*(s)$ has to be on the tangent line of $\boldsymbol{T}(s)$, that is, the target's velocity [see Eq. (2)].

2) Orientation: The target's tangent vector $\boldsymbol{T}(s)$ and the missile's normal vector $\boldsymbol{N}^*(s)$ must be collinear for all s, as C^* intersects the tangent lines of C orthogonally.

3) Speed: The speed should decrease linearly, reaching zero at the impact point: $v^*(s) = (c - s)\kappa$ [see Eq. (3)].

First, restriction 1 is removed using the concept of virtual target described in Sec. III.A. Then problem 2 is solved in Sec. III.B. These developments enable tackling issue 3 by specifying a three-dimensional missile maneuver, which is explained in Sec. III.C.

A. Correct Involute Generated by Virtual Target

A way to overcome restriction 1 is to postulate existence of previous positions of the target, that is, where it could have been before the engagement commenced. This leads to a virtual trajectory obtained by concatenation of the previous positions with the current ones, thus implicitly expanding the observed target trajectory. The expanded trajectory can be traversed by the virtual target at a constant speed, different from the real target's speed. Mathematically, this is an extension of the domain of the target's curve, combined with its reparameterization from s to s_{vrt}. For the real target, Eq. (2) exists on $[0,\ c]$, and hence so do all of the involutes as well. If Eq. (2) is reparameterized by s_{vrt} and extended to $[0,\ c']$, the same happens to its involutes, thus enriching the set of possible missile intercept trajectories.

The virtual target is an imaginary point mass, moving at a constant speed v_{vrt}, which will reach the intercept point at the same time as the real target, but starting at a point behind the real target. (For a forward quarter engagement this should read in front of.) This new starting point is obtained by reparameterizing and extending the real target's trajectory, so that restriction 1 is satisfied for the virtual target. Hence, the missile position at the beginning of the engagement $s = 0$ is on the virtual target's tangent at $s_{\text{vrt}} = 0$.

Assuming that restriction 2 is satisfied, this enables the intercept of the virtual target. Importantly, this can be done by following the trace of the involute the missile is already traversing (at launch). Moreover, this will also lead to the intercept of the real target. Indeed, both targets will arrive at the same time at the point ($s = c$ for the real and $s_{\text{vrt}} = c'$ for the virtual), where the involute trace meets both virtual and real trajectories. Figure 2 illustrates this concept.

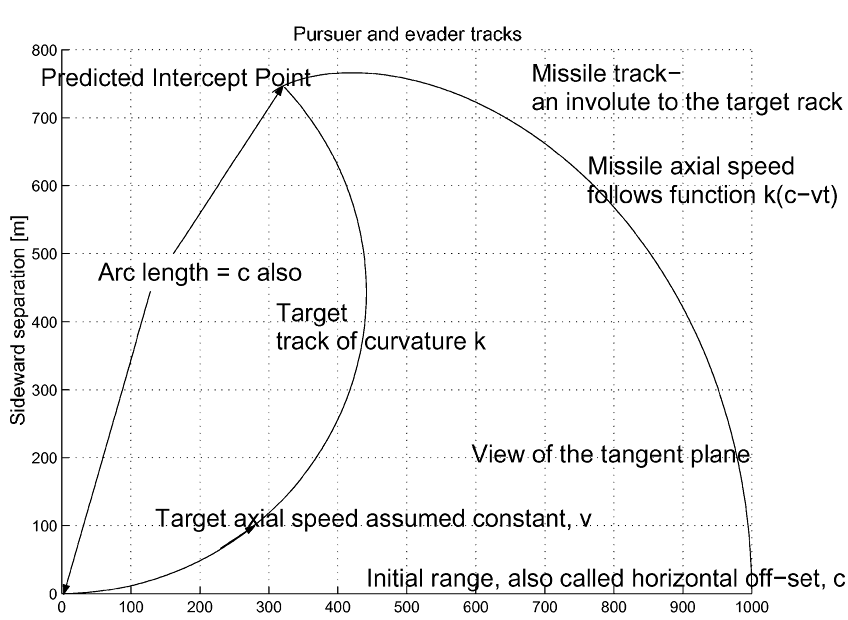

Fig. 1 Trajectory produced by involute guidance law.

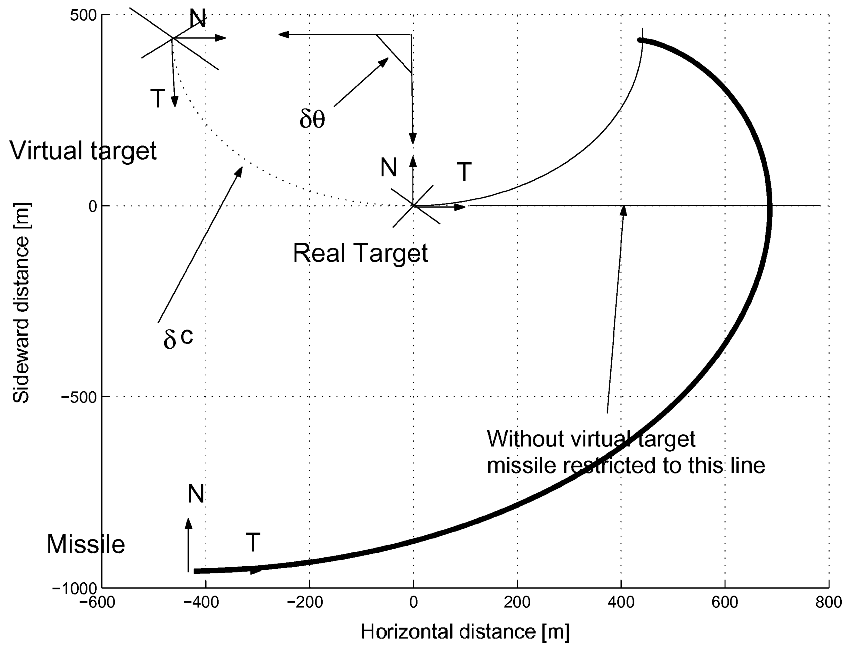

Fig. 2 Finding the correct involute.

To facilitate easy computation of c' and v_{vrt}, we postulate the extension of the observed trajectory by concatenation of a circular arc. The radius of the arc will be computed from the curvature of the target at the beginning of the engagement

$$\rho_{\mathrm{vrt}} = 1/\kappa(0) \tag{6}$$

where $\kappa = \kappa(s)$ is parameterized with s. The virtual trajectory is parameterized by s_{vrt} and at $s_{\mathrm{vrt}} = 0$; its Frenet frame $[\boldsymbol{T}_{\mathrm{vrt}}(0), \boldsymbol{N}_{\mathrm{vrt}}(0)]$ must be orthogonal to the missile's frame at the beginning of the engagement $[\boldsymbol{T}^*(0), \boldsymbol{N}^*(0)]$, that is, at $s = 0$. [Note that the missile's frame is parameterized by s, not s_{vrt}; see Eqs. (2–5).] Because the orientations of both the missile's and the target's frames (both parameterized by s) are assumed to be known at 0, the angular difference between the real and virtual frames $\delta\theta$ is readily computed. Indeed it is the angle through which the target's frame $[\boldsymbol{T}(0), \boldsymbol{N}(0)]$ has to be rotated to be orthogonal to $[\boldsymbol{T}^*(0), \boldsymbol{N}^*(0)]$. Hence the arc length of the extension of the observed trajectory is

$$\delta c = \rho_{\mathrm{vrt}}\, \delta\theta \tag{7}$$

The target moves with the constant speed v and the intercept occurs at $s = c$, so that

$$t_{\mathrm{go}} = c/v \tag{8}$$

whereas, using Eq. (7), we have

$$c' = c + \delta c \tag{9}$$

Hence, the constant speed of the virtual target must be

$$v_{\mathrm{vrt}} = c'/t_{\mathrm{go}} = (1 + \delta c/c)v \tag{10}$$

and, finally,

$$s_{\mathrm{vrt}} = v_{\mathrm{vrt}}\, t = (1 + \delta c/c)\, s \tag{11}$$

which, together with Eq. (9), completes the virtual extension and reparameterization of the target's trajectory. Reparameterizing Eq. (2) with s_{vrt} generates the trace of the correct involute $\mathcal{T}(\boldsymbol{x}^*)$, thus solving problem 1.

The trace is a reference trajectory, but the missile might not arrive at the intercept point at $t_f = t_{\mathrm{go}}$, as it flies at a constant speed. This is the essence of problem 3. If the engagement begins too late, $t_f > t_{\mathrm{go}}$, and the missile will never make it. If the engagement begins too early, $t_f < t_{\mathrm{go}}$, so that it will have to loiter above the tangent plane, while following the trace. This requires a three-dimensional maneuver and is described in Sec. III.C.

B. Alignment Maneuver

As explained in Sec. III.A, the virtual trajectory is obtained by adding a circular arc to the real trajectory. The radius of this arc ρ_{vrt} is given by Eq. (6) and the corresponding arc length δc by Eq. (7), obtained from the angular difference $\delta\theta$ between the real and virtual frames.

Figure 2 shows that for $\delta\theta = \pi/2$ the virtual target's and missile's frames can be aligned, so that restriction 2 is obeyed. If the missile were farther in the negative direction of horizontal separation, there would still exist $\delta\theta > \pi/2$ satisfying requirement 1, but not 2. Although the missile position would then indeed be on the corresponding tangent line of the virtual target, the virtual target's and missile's frames could not be aligned, that is, restriction 2 would be violated. Such a situation is illustrated in Fig. 3, where the initial missile position is $\boldsymbol{x}_m(0) = (0, 4000)$, while the target is at $\boldsymbol{x}(0) = (0, 0)$. The circular arc with radius $\rho_{\mathrm{vrt}} = R_v$ is extended by $-(\frac{3}{2})\pi$, and there exists a tangent line l of this arc on which $x_m(0)$ lies. However, the tangent vector $\boldsymbol{T}_l$ of this line is not collinear with the missile's normal vector, which is $\boldsymbol{N}_m = (0, 1)$, that is, parallel to the direction of sideward separation. If the missile keeps flying straight on (zero maneuver effort), there will come a moment (captured in Fig. 3 for $-\pi/2$) when $\boldsymbol{T}_l$ and $\boldsymbol{N}_m$ are indeed collinear. At that moment the situation is like in Fig. 2, so that the virtual target algorithm can be activated, and the missile will start flying along the correct involute, while obeying restrictions 1 and 2.

Therefore, the two-dimensional aspects of the algorithm are as follows:

Case 1: If, at the beginning of the engagement, the missile is close enough to the target, a virtual target can be computed satisfying 1 and 2. This also means that the missile is immediately on the correct involute and can follow its trace.

Case 2: If, at the beginning of the engagement, the missile is too far for the alignment of its frame with the virtual target's frame to occur, it first executes the alignment phase. This means that the missile continues flying in a straight line until the alignment occurs, that is, when it attains the correct involute and the situation reduces to case 1.

The criterion for distinguishing the cases follows from the definition of the virtual trajectory. The extension of the real trajectory is an

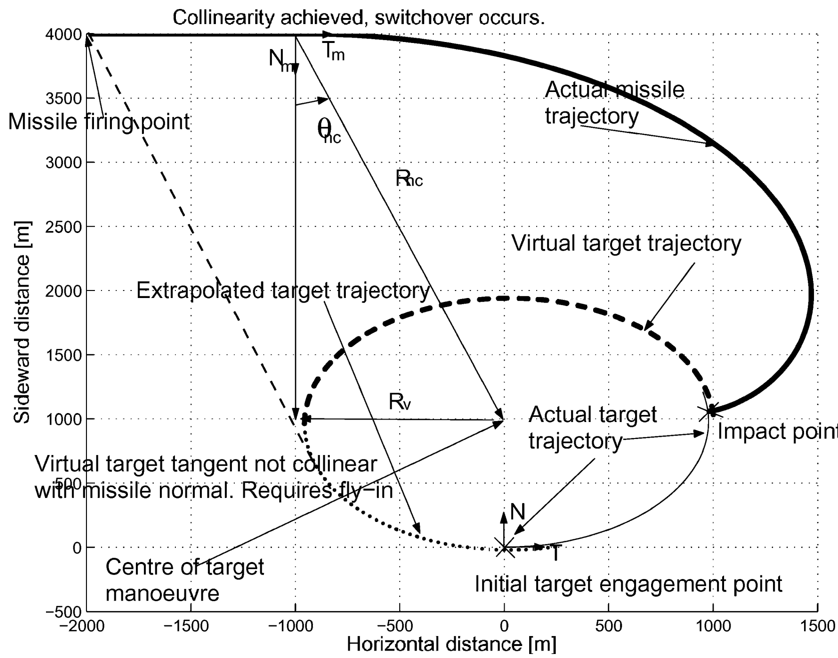

Fig. 3 Forward-quarter target engagement and intercept.

arc of the circle with radius and center derived from the position $\boldsymbol{x}(s)$ and curvature $\kappa(s)$ of the target [see Eq. (6) and Fig. 3]. The criterion tests whether the current normal of the missile is collinear with a tangent of the circle. If this is true, then this is case 1; otherwise, it is case 2.

The vector connecting the target's center of curvature with the missile's position at s (see Fig. 3), is

$$\boldsymbol{R}_{\text{nc}}(s) = [\boldsymbol{x}(s) + \kappa(s)\boldsymbol{N}(s)] - \boldsymbol{x}_m(s) \tag{12}$$

and its length is $R_{\text{nc}}(s) = \|\boldsymbol{R}_{\text{nc}}(s)\|$. The angle between the missile's unit normal vector $\boldsymbol{N}_m(s)$ and $\boldsymbol{R}_{\text{nc}}(s)$ is $\theta_{\text{nc}}(s)$. The candidate circle for the extending arc at s has radius

$$R_v(s) = 1/\kappa(s) \tag{13}$$

Hence, the flying in criterion is

$$\sin\theta_{\text{nc}}(s) \le R_v(s)/R_{\text{nc}}(s) \quad \Rightarrow \quad \text{Case 1}$$

$$\sin\theta_{\text{nc}}(s) > R_v(s)/R_{\text{nc}}(s) \quad \Rightarrow \quad \text{Case 2} \tag{14}$$

The value $s = s_0$ for which case 1 occurs is used to define the virtual target with $\rho_{\text{vrt}} = R_v(s_0)$ [see Eqs. (6) and (13)].

C. Three-Dimensional Missile Maneuver

If problems 1 and 2 are solved, the missile can get to, or is on, the correct involute trace. The trace intercepts the target's trajectory, but the missile must arrive at the intercept point after time to go t_{go}. If the involute length is L_I, then a constant speed missile following the trace will need to fly at $v_I = L_I/t_{\text{go}}$; in general, $v_m \ne v_I$. If $v_m < v_I$, there will be no intercept, but for $v_m > v_I$ the missile can fly above the tangent surface, that is, execute a three-dimensional maneuver. The projection of this three-dimensional trajectory K onto the tangent surface will be the required involute I.

The family of such curves K must satisfy the following requirements: 1) projection of K on the tangent surface is I; 2) K and I coincide at t_{go}; and 3) constant speed v_m missile traverses K in t_{go}.

There are infinitely many curves satisfying the preceding requirements, and they can be found in many ways, for example, using the calculus of variations. However, the focus here is on simple solutions, and therefore the curve K will be sought as a quadric (a fragment of a parabola).

Note that the surface S formed by K, I and the initial vertical offset D is a sector of a developable surface. In other words, S is a fragment of the surface generated by straight lines orthogonal to the tangent surface T and moving along I. Development of S means that it can be considered in the $[s,\ z(s)]$ plane, where s is the arc length of I and $z(s)$ the height of K above T. For a planar target T is a plane, and thus S is a cylindrical sector. From now on we shall focus on this case, assuming that T is the $(x,\ y)$ plane. In practice, this means that the three-dimensional maneuver curve K will have the same x and y coordinates as the involute I, but its z coordinate will be a quadric in s, the target's arc length:

$$z(s) = As^2 + Bs + C \qquad A, B, C \in \mathbb{R} \tag{15}$$

Recall that the target's arc length s is related to time through $s = vt$, where v is the target's speed.

To determine A, B, C ensuring the intercept, we need first to determine the length L_I of the involute I. Using Eq. (3), it follows that for a target with a constant curvature κ

$$L_I = \int_0^c \|\boldsymbol{v}^*(s)\|\,\mathrm{d}s = \frac{1}{2}\kappa c^2 \tag{16}$$

where $\kappa = \text{const}$ is measured for the parameterization with respect to s. If the constant curvature is expressed with respect to time t, it will have value $\kappa_t = \kappa v$, so that $L_I = \kappa_t c^2/(2v)$.

Note that $t_{\text{go}} = c/v$ and the length of K is

$$L_K = \int_0^c \sqrt{1 + \left(\frac{\mathrm{d}z}{\mathrm{d}s}\right)^2}\,\mathrm{d}s \tag{17}$$

so that, according to Eq. (15),

$$z(0) = D \tag{18}$$

$$z(c) = L_I \tag{19}$$

$$\int_0^c \sqrt{1 + (2As + B)^2}\,\mathrm{d}s = v_m\, t_{\text{go}} \tag{20}$$

where Eq. (20) means that the length L_K of K must be traversed by the missile in time to go. The indeterminate integral $\int \sqrt{[1 + (2As + B)^2]}\,\mathrm{d}s$ has a closed form, which is not necessarily simple, so that in actual implementation Eq. (20) was solved iteratively as follows. From Eq. (18) we have that $C = D$, while Eq. (19) gives

$$A = -(Bc + D)/c^2 \tag{21}$$

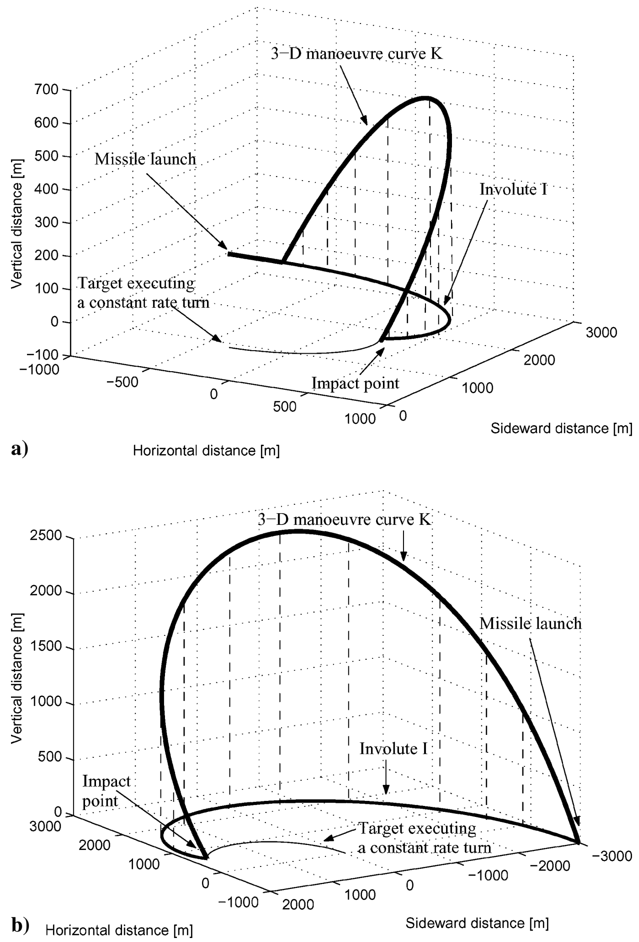

Fig. 4 Three-dimensional maneuver induced by the involute law for planar target: a) forward-quarter engagement and b) rear-quarter engagement.

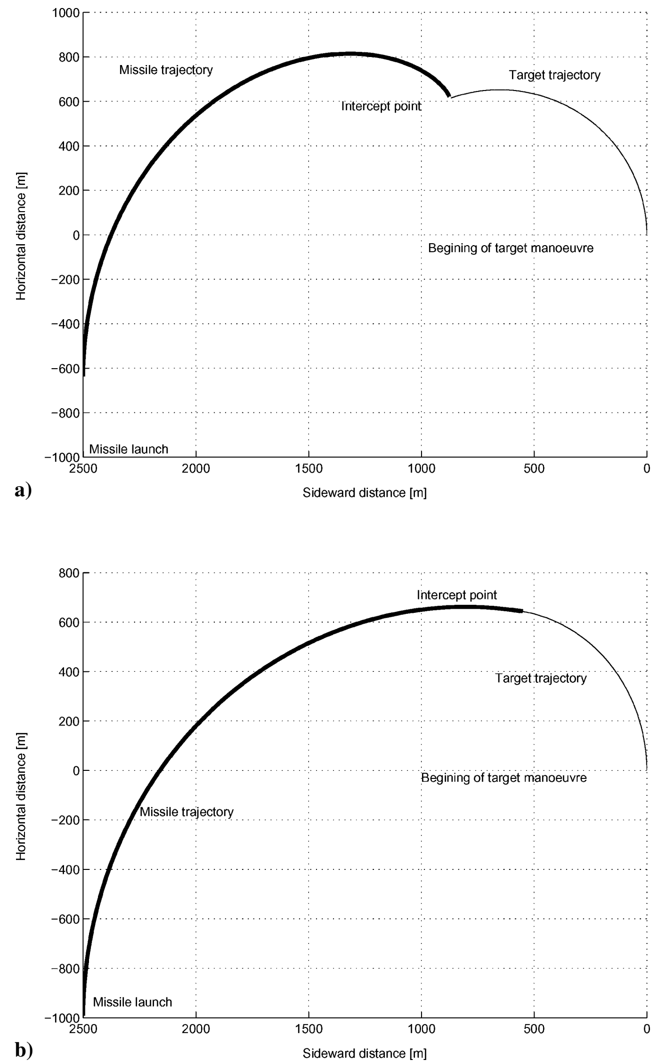

Fig. 5 Forward-quarter engagement for planar target: a) involute law (projection onto the tangent plane; compare with Fig. 4a) and b) PN law.

If B is found, so that Eq. (20) is satisfied with required accuracy, Eq. (21) can be used readily to compute A, and Eq. (15) is determined, guaranteeing intercept. Because B is the tangent of the quadric at $s = 0$, or $z = D$, iterative solution of Eq. (20) is done by simple shooting with respect to $\alpha \overset{\text{def}}{=} \operatorname{atan}(B)$, where $-\pi/2 < \alpha < \pi/2$. Corrections to the shooting are done by the bisection algorithm on $(-\pi/2,\ \pi/2)$ to ensure rapid convergence.

IV. Simulated Scenarios and Results

The following assumptions¶ are made regarding the nature and capabilities of the target: 1) the target is an F-16 class fighter; 2) the target will maneuver in the endgame; 3) maneuver up to 9 g, sustainable for < 10 s; and 4) the missile-target engagements are all aspect.

A. Planar Engagement: Constant g-Turn Maneuver

The evasive maneuver assumed here is a tightly banked turn or loop.[11]

The preceding capabilities will now be described in terms of differential geometric parameters, namely, $\boldsymbol{T}, \boldsymbol{N}, \kappa$. The principal plane of the maneuver, or the tangent plane, is horizontal in the Earth Cartesian coordinate system. In this context, the following assumptions are made for the tightly banked turn maneuver: 1) the curvature of the target's trajectory is known; 2) the curvature is constant $\kappa = 0.34$; 3) the turn is less than one complete cycle; and 4) the target's speed is 300 m/s (equivalent air speed). Two engagements are simulated: 1) forward-quarter engagement (Figs. 4a and 5), and 2) rear-quarter engagement (Figs. 4b and 6).

In the simulations it was assumed that, at the beginning of the engagement, the missile is on the tangent plane, so that initial offset $D = 0$.

For performance comparison, the two-dimensional results (projections) of the involute law are contrasted with the simulation results for the PN law. In the case of forward engagement, Fig. 5 for the involute law is compared with the PN law; for rear engagement, see Fig. 6.

Both the PN (which is a benchmark) and the new algorithm intercept the target for perfect target information. However, the engagement times and trajectories are different.

The involute engagement model guarantees a perpendicular impact in the tangent plane, or side impact in three dimensions. The PN guidance law varies in the impact angle, depending on the engagement. In the two scenarios displayed, it evolves into a head-on or tail-chase maneuver. In general, the benchmark provides a shorter engagement time, as it takes a more direct approach to the intercept point.

B. Spatial Engagement: Barrel-Roll Maneuver

Barrel-roll maneuver is challenging for intercept, because 1) it produces an oscillating line of sight and 2) it involves out-of-plane maneuvers.

¶Data available online at http://www.fas.org/man/dod-101/sys/ac/f-16.htm.

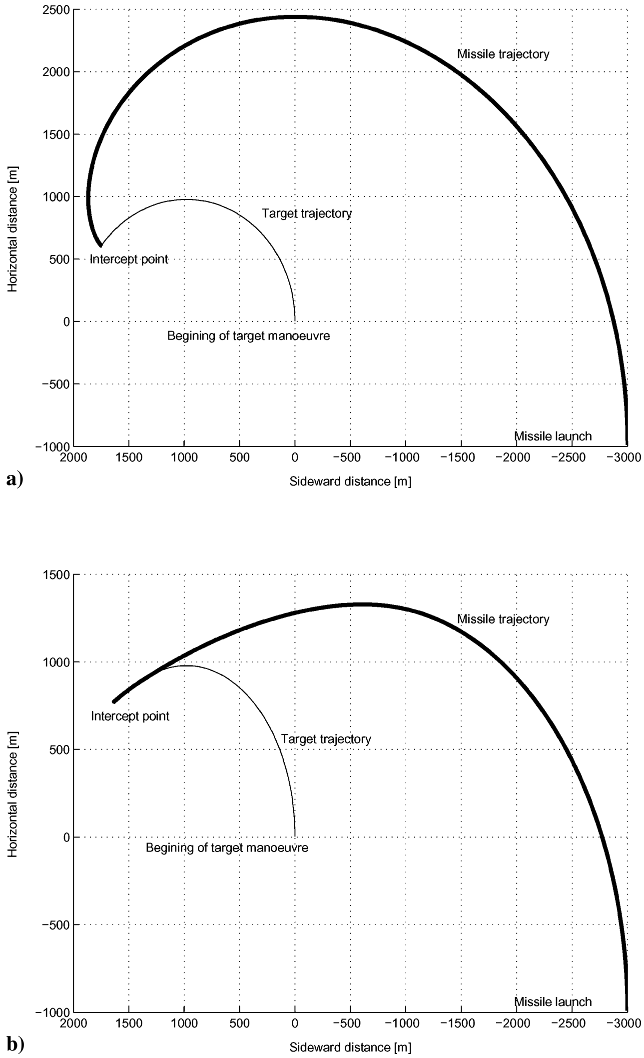

Fig. 6 Rear-quarter engagement for planar target: a) involute law (projection onto the tangent plane; compare with Fig. 4b) and b) PN law.

Mathematically, the trajectory of a barrel-rolling target can be approximated by a circular helix with radius $a > 0$ (Ref. 3, p. 85):

$$\boldsymbol{x}(t) = [a\cos(t), a\sin(t), bt] \tag{22}$$

whose involute $\boldsymbol{x}^*(s) = \boldsymbol{x}(s) + (c - s)\boldsymbol{t}(s)$ parameterized by $t = s/\sqrt{(a^2 + b^2)}$ is

$$\boldsymbol{x}^*(t) = \{a[\cos(t) + t\sin(t) - \gamma\sin(t)], a[\sin(t) - t\cos(t) + \gamma\cos(t)], b\gamma\} \tag{23}$$

where $\gamma = c/\sqrt{(a^2 + b^2)}$. Thus, Eq. (23) is a plane curve, lying in the plane $x_3 = b\gamma$.

The speed of the target in the barrel-roll maneuver is $v = \sqrt{(a^2 + b^2)}$, its speed along the x_3 axis is $v_{\mathrm{ax}} = b$, and the speed projected on the tangent plane is $v_{\mathrm{proj}} = a$. Because both the helix curvature and torsion are constant, $\kappa = a/(a^2 + b^2)$ and $\tau = b/(a^2 + b^2)$, we have $v_{\mathrm{ax}}/v_{\mathrm{proj}} = \tau/\kappa$. This ratio can be used to quantify the rapidity of the barrel roll, that is, how far the target progresses along the x_3 axis in each cycle (time interval of 2π).

Based on the F-16 capabilities, typical values for a barrel-roll maneuver's parameters are assumed to be the following: 1) the target curvature κ is between 0 and 0.285 s^{-1}; 2) the target torsion τ is between one and five times curvature; 3) the target axial speed v_{ax} is approximately 310 ms^{-1} EAS; and 4) the roll lasts between one and three cycles.

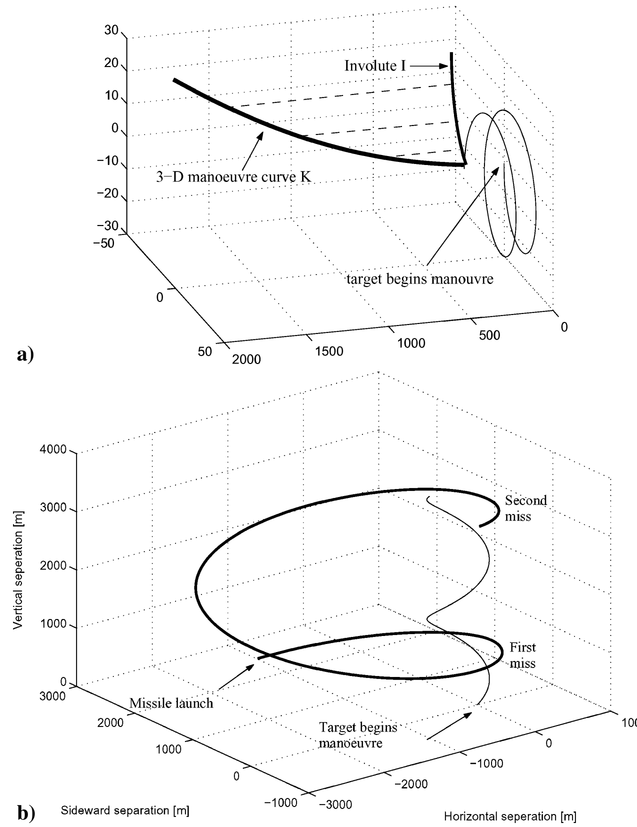

Fig. 7 Forward-quarter engagement for target in barrel-roll: a) involute law and b) PN law.

As can be seen from Fig. 7b, the PN law cannot cope with the maneuver. By contrast, the new law achieves intercept (see Fig. 7a). This happens because the involute of the target is a planar curve, so that—from the intercept algorithm viewpoint—the situation is no different from a planar engagement. Indeed, the derivations presented in Sec. III were based on the assumption that the target's involute is planar, which is true for barrel-roll represented as a helix.

V. Conclusions

This paper presents a basic engagement model for a maneuvering target, derived from the differential geometric concept of involute. Assuming that the target's curvature and torsion are available (effective estimators for which are yet to be derived), the resulting guidance law is shown to be viable and simple and does not rely on line-of-sight (LOS) information.

Compared with proportional navigation (PN), the missile trajectories generated by the involute law have unexpected, curved shapes. Although this results in longer engagement times than PN for planar engagements, it adds uncertainty to the evasion strategy, especially that pilots tend to base the strategy on the LOS information. Furthermore, a side impact angle means that the missile is approaching the target from its blind side, if it is pulling maximum g in a level turn, increasing lethality. Unlike PN, the new law automatically works against barrel-roll maneuver.

References

[1] Ben-Asher, J. Z., and Yaesh, I., *Advances in Missile Guidance Theory*, AIAA, Reston, VA, 1998.

[2] Shneydor, N. A., *Missile Guidance and Pursuit, Kinematics, Dynamics and Control*, Horwood Publishing, Chichester, England, U.K., 1998, Chap. 9.

[3] Lipschutz, M. M., *Differential Geometry*, Schaum's Outline Series, McGraw–Hill, New York, 1969, Chap. 5.

[4] Chiou, Y. C., and Kuo, C. Y., "Geometric Approach to Three-Dimensional Missile Guidance Problem," *Journal of Guidance, Control, and Dynamics*, Vol. 21, No. 2, 1998, pp. 335–341.

[5]Kuo, C. Y., and Chiou, Y. C., “Geometric Analysis of Missile Guidance Command,” *IEE Proceedings: Control Theory and Applications*, Vol. 147, No. 2, 2000, pp. 205–211.

[6]Kuo, C. Y., Soetanto, D., and Chiou, Y. C., “Geometric Analysis of Flight Control Command for Tactical Missile Guidance,” *IEEE Transactions on Control Systems Technology*, Vol. 9, No. 2, 2001, pp. 234–243.

[7]Lin, C.-F., *Modern Navigation Guidance and Control Processing*, Prentice–Hall, Upper Saddle River, NJ, 1991, Chap. 8.

[8]Serakos, D., and Lin, C.-F., “Linearized Kappa Guidance,” *Journal of Guidance, Control, and Dynamics*, Vol. 18, No. 5, 1995, pp. 975–980.

[9]Serakos, D., and Lin, C.-F., “Three Dimensional Mid-Course Guidance State Equations,” *Proceedings of the 1999 American Control Conference*, Vol. 6, American Automatic Control Council, San Diego, CA, 1999, pp. 3738–3742.

[10]Cherry, G. W., “A General Explicit, Optimizing Guidance Law for Rocket-Propellant Spacecraft,” AIAA Paper 64-638, 1964.

[11]Shaw, R. L., *Fighter Combat. The Art and Science of Air-to-Air Warfare*, 2nd ed., Patrick Stephens, Wellingborough, England, U.K., 1988.